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Wilson-Polyakov surfaces and M-theory branes

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Abstract: In this paper, we study the M-brane description of the Wilson-Polyakov surfaces in six-dimensional $(2,0)$ field theory at finite temperature. We investigate the membrane solution dual to a straight Wilons-Polyakov surface and compute the interaction potential between two parallel straight strings by using AdS/CFT correspondence. Furthermore we discuss the M5-brane solutions dual to various Wilson-Polyakov surfaces. Finally we obtain a universal result about M5-brane solutions in generic backgrounds.

Keywords: AdS-CFT Correspondence, p-branes, M-Theory.

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Contents

1. Introduction

The six-dimensional (2,0) superconformal field theory (SCFT) is mysterious, but also very interesting. From quantum field theory point of view, it has been known for many years that there exist no non-trivial unitary superconformal field theory in the spacetime higher than four dimension, with the six-dimensional one being the only exception. The existence of a six-dimensional superconformal field theory was first pointed out in Nahm's beautiful paper [1]. The result was obtained by studying the representation of superconformal algebra in various dimensions [1]. The same issue was readdressed in [2] from the point of view of scaling invariance. The superconformal field theory in six-dimensional spacetime has $(2,0)$ supersymmetries and its field content is just of a tensor multiplet which includes a two-form $B_{\mu\nu}$ with self-dual field strength, 4 fermions and 5 scalars. Because of the self-dual two form field, there is no Lagrangian formulation of this quite mysterious theory [3], although this theory is still a local interacting field theory [4]. After compactified on a two-torus, the six-dimensional field theory gives us the four-dimensional $\mathcal{N}=4$ super Yang-Mills theory at the low energy limit [5, 3]. This fact can be used to study the properties of this very notable four dimensional superconformal field theory, such as S-duality. In string theory, this six-dimensional SCFT appears in several contexts. It appears when we consider IIB string theory on a K3 surface with A-D-E type singularity [5] and also appears as the low energy effective field theory of M5-branes [6, 7]. In the latter case, if we consider N M5-branes on top of each other, the low energy effective field theory is the six-dimensional $A_{N-1}, (2,0)$ SCFT. The five scalars in this field theory describe the fluctuations of the M5-branes in the transverse directions. Since there is no Lagrangian formulation of the theory,

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people tried to study this theory from other angles. A DLCQ matrix model description of the six-dimensional superconformal field theory has been suggested [8, 9], during the development of the BFSS matrix theory [10].

There is another description of M5-branes [11]: they can be described as solutions of eleven dimensional supergravity, which is the low energy effective theory of M-theory. Taking the near horizon limit of the supergravity solution gives us $AdS_7 \times S^4$ background with 4-form flux filling in S^4. This led Maldacena to propose the conjecture that the M-theory on $AdS_7 \times S^4$ is dual to the large N limit of the six dimensional superconformal field theory [12]. This AdS_7/CFT_6 correspondence is a cousin of much more well-known AdS_5/CFT_4 correspondence proposed in the same paper. The weak version of the above AdS_7/CFT_6 correspondence states that the large N limit of six-dimensional $(2,0)$ SCFT is dual to the eleven-dimensional supergravity on $AdS_7 \times S^4$. This correspondence gives us a new way to study the six-dimensional theory. The chiral primary operators of the SCFT and the corresponding supergravity modes were studied in [13]. Some correlation functions of these local operators were computed in [14] from AdS supergravity. These operators were also studied by using M5-brane action in [15].

Non-local operators play important roles in AdS/CFT correspondence. In the AdS_5/CFT_4 correspondence, the Wilson loops in fundamental representation or low dimensional representation can be described using the fundamental strings [16, 17]. However, it turns out that the better descriptions of the BPS Wilson loops in higher dimensional representations are in terms of D-branes in $AdS_5 \times S^5$ [18 − 21] due to dielectric effect [22]: D3-branes if the Wilson loops being in symmetric representations, or D5-branes if being in antisymmetric representations. The D-brane description of Wilson-'t Hooft operators was discussed in [23]. It is remarkable that the D-branes description of the BPS Wilson loops encodes the information of string interactions.

A quite similar picture appears also in the AdS_7/CFT_6 correspondence. Due to existence of the self-dual 2-form potential, there are strings in the field theory minimally coupled to the 2-form potential. This allows us to define a two-dimensional non-local operator called Wilson surface. It can be formally defined as [24]:

$$W_0(\Sigma) = \mathrm{Tr}\left(\exp i \int_\Sigma B^+\right), \tag{1.1}$$

where Σ is a surface in the six-dimensional spacetime. The Wilson surface in low dimensional representations is dual to a membrane ending on this surface [17, 25]. The M5-branes dual to straight and spherical half-BPS Wilson surfaces in higher dimensional representation were found in [26] by solving the covariant equations of motions for M5-branes[1]. Analogues to the Wilson loop case, the worldvolume of the M5-brane dual to the Wilson surface in symmetric representation has topology $AdS_3 \times S^3$ and is completely embedded in AdS_7. While the worldvolume of the M5-brane corresponding to the Wilson surface in antisymmetric representation has the same topology but with the S^3 part in S^4. The expectation value of the Wilson surfaces in higher dimensional representation

[1]Similar M5-brane configurations for straight Wilson surface are discussed in [27] in Pasti-Sorokin-Tonin (PST) formalism [28, 29] as well. The self-dual string soliton in $AdS_4 \times S^7$ spacetime is discussed in [27, 30].

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was computed in [26] from the action of M5-brane, without including the subtle boundary terms. The operator product expansion of Wilson surface operators is computed using M-theory branes in [14, 31].

The Wilson loop or Wilson surface operators are not just probes to test AdS/CFT correspondence and probe the strings or membranes dynamics. They are physical gauge invariant observables to characterize the underlying theory. For example, in pure non-Abelian gauge theory at finite temperature, the Wilson loop along temporal path, the so-called Wilson-Polyakov loop, defined by

$$P(\vec{x}) = \frac{1}{N}\mathrm{Tr}\left(\mathcal{P}\exp\left(i\int_0^\beta A_0(\vec{x})dt\right)\right), \tag{1.2}$$

is the order parameter, characterizing the phase of the theory. Here $\beta = 1/T$ is the inverse temperature and $\mathcal{P}$ denotes the path ordering. Moreover by considering the correlator of two parallel Wilson-Polyakov loops one can read out the static potential between two quarks.

The AdS_5/CFT_4 correspondence has a finite temperature extension. At finite temperature, The spacetime where the four-dimensional field theory lives can be either $S^3 \times S^1$ or $R^3 \times S^1$. Now the time direction becomes a circle with the period being the inverse of the temperature. According to the AdS/CFT correspondence, this finite temperature theory is dual to type IIB string theory on the background which is the product of a Schwarzschild black hole in AdS_5 space[2] and a 5-sphere. This background comes from the near-horizon limit of non-extremal black 3-brane solution of the type IIB supergravity. The Hawking temperature of the black hole corresponds to the temperature of the field theory. In [32, 33], Witten studied the thermodynamics of this theory on $S^3 \times S^1$. He showed that there is confinement-deconfinement phase transition in this theory. This transition is dual to the Hawking-Page transition [34] in the gravity side. In this case, the AdS/CFT correspondence tells us that the Wilson-Polyakov loops in fundamental representation can be described by fundamental strings in $Sch.$-AdS_5 space [35]. Using this description the interaction potential between two heavy quarks at finite temperature was studied. The field theory is on $R^3 \times S^1$ in [35], so there is no confinement-deconfinement phase transition. One may expect that for the Wilson-Polyakov loop in higher representation, D-branes rather than fundamental string are more appropriate. In [36], the D5-brane description of the Wilson-Polyakov loops was proposed. It was also showed in that paper that there is no D3-brane description. In [37], the correlation function of two Wilson-Polyakov loops, one in fundamental representation and the other one in the anti-symmetric representation, was computed.

Similarly the six-dimensional $(2,0)$ field theory at finite temperature is dual to M-theory on $Sch.$-$AdS_7 \times S^4$. This background can come from the the near horizon limit of non-extermal black M5-brane solution. The Hawking temperature in the gravity side is still corresponding to the temperature in the field theory side. The gravity dual of this finite temperature theory was used in [33] to study the nonsupersymmtric pure Yang-Mills theory in four dimensions.

[2]We denote this by $Sch.$-AdS_5.

In this paper, we would like to study the counterpart of the Wilson-Polyakov loop in finite temperature six-dimensional $(2,0)$ field theory in $R^5 \times S^1$ using the M-theory branes. Like the Wilson surface operator, this counterpart should be a two-dimensional non-local operator. We also expect that it extends in one spatial direction and one temporal direction. We can still formally define this operator using eq. (1.1), while now Σ should be a surface on which the induced metric has signature $(1,1)$. We call this operator Wilson-Polyakov surface.

We propose that when this operator is in lower dimensional representations it should be described by M2-branes ending on this surface. We first find the membrane solution corresponding to a straight Wilson-Polyakov surface. We also compute the potential between two static parallel self-dual strings with infinite length. This involves two Wilosn-Polyakov surfaces extending in the same two directions of the spacetime. The interaction potential between these two strings can be obtained from the correlation function of these two Wilson-Polyakov surfaces. There are two classes of membrane configurations ending on these two Wilson-Polyakov surfaces: one is of two separated membranes, each of which ending on one Wilson-Polyakov surface; the other one is a U-shape membrane connecting these two Wilson-Polyakov surfaces. The interaction potential is determined by the lowest energy configuration. Denoting the distance between two strings as L and the temperature as T, we find that when $LT \ll 1$, the potential per length goes as $1/L^2$, similar to the results at zero temperature [17], and when $LT \gg 1$, the interaction potential vanish since it is screened by the thermal effects.

We also study the M5-brane configurations which should be dual to the Wilson-Polyakov surfaces in higher dimensional representations. Similar to the discussions at zero temperature, two classes of M5-branes are studied. The first class of the M5-brane is completely embedded in $Sch.\text{-}AdS_7$, while the second class of M5-brane has a S^3 part embedded in S^4. In the first case, we get a very complicated differential equation. The existence of the solution of this equation is discussed. While in the second case, we obtain a class of explicit solutions. Among these solutions, we indicate a special one which should be dual to the Wilson-Polyakov surface in the anti-symmetric representation.

Furthermore, inspired by the M5-brane solution dual to the Wilson-Polyakov surface in antisymmetric representation and the similar solution at zero temperature in [26], we consider M-theory on $M_7 \times S^4$ which can be dual to a quite generic field theory at the boundary of M_7. We obtain a universal result on M5-brane solutions in $M_7 \times S^4$. Starting with a membrane solution whose worldvolume Σ_3 is completely embedded in M_7, we find that there is always an M5-brane solution whose topology is $\Sigma_3 \times S^3$ with the same Σ_3 in AdS_7 and S^3 in S^4. The similar universal result for D5-brane solutions corresponding to Wilson loops in anti-symmetric representation was discussed in [38].

The investigation we make here may help us to get a better understanding of the six-dimensional field theory at finite temperature, the dynamics of the M-theory branes, and even the dynamics of M-theory itself. For a very good review of the dynamics of M-theory branes, see [39].

The other part of this paper is organized as the following: In section 2, we will discuss the M2-brane description of the Wilson-Polykov surface. Firstly in subsection 2.1, the

M2-brane dual to a straight Wilson-Polykov surface is discussed, then in subsection 2.2, the interaction between two self-dual strings is studied using M2-brane description. Our discussions on the M5-brane description is put in section 3. In subsection 3.1, we investigate the M5-brane which is completely embedded in $Sch.\text{-}AdS_7$, in subsection 3.2, we discuss the M5-brane solution with an S^3 part in S^4, and in subsection 3.3, we present the universal result we find. The last section is devoted to conclusion and discussions.

2. Membrane description

As mentioned before, the six-dimensional $(2,0)$-field theory at finite temperature is believed to be dual to the M-theory on $Sch.\text{-}AdS_7 \times S^4$. We also have a 4-form flux which fills in the 4-sphere. The metric of the background is

$$ds^2 = \frac{R^2}{y^2}\left(\frac{dy^2}{f(y)} - f(y)dt^2 + \sum_{i=1}^{5} dx_i^2\right) + \frac{R^2}{4}d\Omega_4^2, \tag{2.1}$$

with

$$f(y) = 1 - \epsilon^6 y^6. \tag{2.2}$$

Here $d\Omega_4^2$ is the metric of unit 4-sphere. If $\zeta_i, i = 1, \cdots, 4$, are the angular coordinates of the 4-sphere, then $d\Omega_4^2$ can be written as:

$$d\Omega_4^2 = d\zeta_1^2 + \sin^2\zeta_1 d\zeta_2^2 + \sin^2\zeta_1 \sin^2\zeta_2 dz_3^2 + \sin^2\zeta_1^2 \sin^2\zeta_2^2 \sin^2\zeta_3^2 d\zeta_4^2. \tag{2.3}$$

The background 4-form field strength is

$$H_4 = \frac{3R^3}{8}\sin^3\zeta_1 \sin^2\zeta_2 \sin\zeta_3 d\zeta_1 \wedge d\zeta_2 \wedge d\zeta_3 \wedge d\zeta_4. \tag{2.4}$$

This background can be obtained from the near-horizon limit of non-extremal black M5-brane solution of the 11-dimensional supergravity. From the AdS/CFT correspondence, the relation among R, 11-dimensional Plank length l_p and the parameter N is

$$R = (8\pi N)^{\frac{1}{3}} l_p. \tag{2.5}$$

In the large N limit, this six-dimensional field theory at finite temperature should be dual to the 11-dimensional supergravity in this background. In the metric (2.1), the boundary field theory is defined at the conformal infinity where $y = 0$, and the coordinates on the boundary are $t, x_i, i = 1, \cdots, 5$. The topology of the boundary is $R^5 \times S^1$ with S^1 in the time direction.

Using the standard method, we can get the Hawking temperature T_H of the black hole:

$$T_H = \frac{3}{2\pi}\epsilon. \tag{2.6}$$

This Hawking temperature corresponds to the temperature of the field theory.

In this paper, we will use AdS/CFT correspondence to study the Wilson-Polyakov surface operators in the six-dimensional field theory at finite temperature. In this section, we will study the M2-brane description of the Wilson-Polyakov surface operators.

The bosonic part of the membrane action is[3] [40]

$$S_{M2} = T_2 \left(\int d^3\xi \sqrt{-\det g_{\mu\nu}} - \int \underline{C_3} \right), \tag{2.7}$$

where g_{mn} is the induced metric on the membrane, T_2 is the tension of M2-brane:

$$T_2 = \frac{1}{(2\pi)^2 l_p^3}, \tag{2.8}$$

and $\underline{C_3}$ is the pullback of the bulk 3-form gauge potential to the worldvolume of the membrane[4]. The membrane equations of motions are:

$$\frac{1}{\sqrt{-g}}\partial_m \left(\sqrt{-g} g^{mn} \partial_n X^{\underline{N}}\right) G_{\underline{MN}} + g^{mn} \partial_m X^{\underline{N}} \partial_n X^{\underline{P}} \Gamma^{\underline{Q}}_{\underline{NP}} G_{\underline{QM}} = \frac{1}{3!} \epsilon^{mnp} \underline{H}_{\underline{M}mnp}. \tag{2.9}$$

Here $\underline{\mathrm{H}}$ is the pullback of the background four-form field strength.

2.1 Membrane description of Wilson-Polyakov surface

In this subsection, we consider a straight Wilson-Polyakov surface in six-dimensional $(2,0)$ theory at finite temperature. We let it extend in the t and x_2 directions. The dual membrane configuration is always ending on this Wilson-Polyakov surface. This means that two worldvolume coordinates of membrane could be identified with t and x_2, while the other coordinate should extend into the bulk.

The simplest membrane configuration is to let it extend only along y direction. We can easily check that this membrane configuration is the solutions of the membrane equations of motion. The only non-trivial equation is the one with index $\underline{M} = y$, which can be checked by straightforward calculations. The needed Christoffel symbol of the $Sch.$-AdS_7 space is listed in appendix.

The membrane should be stretched between the boundary ($y = 0$) and the horizon ($y = y_0 \equiv 1/\epsilon$) because the non-extremal N black M5-branes should be located at the horizon. Arguments supporting similar result in the $Sch.$-AdS_5 case can be found in [41, 35]. Some of these arguments can be applied here. The action of this membrane is

$$\begin{aligned} S &= T_2 R^3 T_0 X_2 \int_0^{y_0} \frac{dy}{y^3} \\ &= \frac{2N}{\pi} T_0 X_2 \int_0^{y_0} \frac{dy}{y^3}. \end{aligned} \tag{2.10}$$

where T_0, X_2 are the lengths of the t, x_2 direction, and in the second line of the above equation, $T_2 R^3 = 2N/\pi$ is used. By introduce a cutoff $y = \delta$ near $y = 0$, we get,

$$S = -\frac{N}{\pi} T_0 X_2 \left(\frac{1}{y_0^2} - \frac{1}{\delta^2} \right). \tag{2.11}$$

[3]Our notation is: the indices from the beginning(middle) of the alphabet refer to the frame (coordinate) indices, and the underlined indices refer to the target space ones.

[4]In this paper, we use the underline indices to denote the target space indices. We also use the underline to denote the pullback of bulk gauge potential or field strength to the worldvolume of M2-brane or M5-brane. We hope that this will not produce confusion.

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2.2 The potential between two static strings

It would be interesting to study the potential between two static parallel infinitely-long strings in this six-dimensional finite-temperature field theory. We let these strings extend along the x_2 direction and put them at $x_1 = L/2$ and $x_1 = -L/2$. To compute this potential, we need to consider two Wilson-Polyakov surfaces. These surfaces should extend along the t and x_2 direction and be put at $x_1 = L/2$ and $x_1 = -L/2$, respectively. [5]

There are two classes of membrane configurations ending on these two Wilson-Polyakov surfaces. The first is two separated parallel membranes, with each membrane ending on one of the two Wilson-Polyakov surfaces and extending along y direction to the horizon. These membranes and their actions have been studied in the last subsection. The second is a U-shape membrane connecting these two Wilson-Polyakov surfaces. Now we will study this membrane solution. In the large N limit, the potential between these two static strings will be determined by the lowest energy membrane configuration among the possible classical solutions.

The connected membrane solution will only extend in x_1, x_2, t, y directions of the background geometry. We choose the coordinates of the worldvolume of the corresponding membrane to be x_1, x_2, t, and y is a function of x_1 only. Then boundary condition is: $y(-L/2) = y(L/2) = 0$.

The induced metric on this membrane is

$$ds^2_{\text{ind}} = \frac{R^2}{y^2}\left(-f dt^2 + dx_2^2 + \left(1 + \frac{y'^2}{f}\right)dx_1^2\right), \tag{2.12}$$

where y' is dy/dx_1. Then the action of this membrane is,

$$S = T_2 \int dt dx_1 dx_2 \sqrt{-g} = T_2 R^3 T_0 X_2 \int \frac{\sqrt{f + y'^2}}{y^3} dx_1, \tag{2.13}$$

where T_0, X_2 are the lengths of the t, x_2 direction.

We can consider the above action as the one of an imaginary particle with x_1 plays the role of time. Since the Lagrangian does not depend on x_1 explicitly, the Hamiltonian in the x_1 direction is a constant of motion. So

$$p_y y' - \mathcal{L} = -\frac{T_2 R^3 T_0 X_2 f}{y^3\sqrt{f + y'^2}} = const. \tag{2.14}$$

According to the symmetries of this system, at $x_1 = 0$, y should reach its maximum value y_m. So at this point, $y' = 0$. Note that we only consider the membrane solution out of the horizon, so $y_m \leq y_0$. Using the above equation, we have

$$\frac{f(y)}{y^3\sqrt{f(y) + y'^2}} = \frac{f(y_m)}{y_m^3\sqrt{f(y_m)}} = \frac{\sqrt{f(y_m)}}{y_m^3}, \tag{2.15}$$

so

$$y'^2 = \frac{(1 - \epsilon^6 y^6)(y_m^6 - y^6)}{y^6(1 - \epsilon^6 y_m^6)}. \tag{2.16}$$

[5]We choose the same scalar coupling for these two Wilson-Polyakov surfaces, in another word, we choose the same point at S^4 for them.

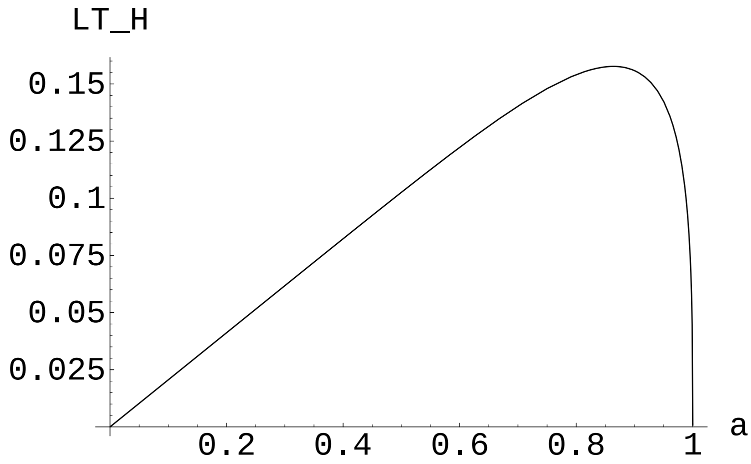

Figure 1: The functional relation of LT_H and a. One can see that for $L < L_{\max}$, there are two connected membrane solutions, while for $L > L_{\max}$, there are no connected membrane solutions.

We can solve this equation to get:

$$x_1 = \begin{cases} \int_y^{y_m} \sqrt{\frac{1-\epsilon^6 y_m^6}{(1-\epsilon^6 z^6)(y_m^6 - z^6)}} z^3 dz, & \text{when } x_1 > 0; \\ -\int_y^{y_m} \sqrt{\frac{1-\epsilon^6 y_m^6}{(1-\epsilon^6 z^6)(y_m^6 - z^6)}} z^3 dz, & \text{when } x_1 < 0. \end{cases} \tag{2.17}$$

Because of the boundary conditions, we have the following relations between y_m and L:

$$L = 2 \int_0^{y_m} \sqrt{\frac{1-\epsilon^6 y_m^6}{(1-\epsilon^6 z^6)(y_m^6 - z^6)}} z^3 dz. \tag{2.18}$$

By introducing $a \equiv \epsilon y_m$, L can be written as:

$$L = \frac{2a}{\epsilon} \sqrt{1-a^6} \int_0^1 \frac{z^3 dz}{\sqrt{(1-z^6)(1-a^6 z^6)}} \tag{2.19}$$

The above result can be expressed by the hypergeometric function as the following:

$$L = \frac{2\sqrt{\pi}\Gamma(2/3)a}{\epsilon\Gamma(1/6)} \sqrt{1-a^6}\, {}_2F_1\left(\frac{1}{2}, \frac{2}{3}, \frac{7}{6}, a^6\right). \tag{2.20}$$

From this, we can also get the dimensionless combination LT_H as a function of a:

$$LT_H = \frac{3\Gamma(2/3)a}{\sqrt{\pi}\Gamma(1/6)} \sqrt{1-a^6}\, {}_2F_1\left(\frac{1}{2}, \frac{2}{3}, \frac{7}{6}, a^6\right). \tag{2.21}$$

This function is plotted in figure 1.

One can see that L has a maximal value $L_{\text{max.}}$. For each L less than $L_{\text{max.}}$, there are two corresponding a's: $a_1(L)$ and $a_2(L)$ with $a_1(L) < a_2(L)$. So there are two connected membrane configurations. For $L > L_{\text{max.}}$, there are no connected membrane solutions.

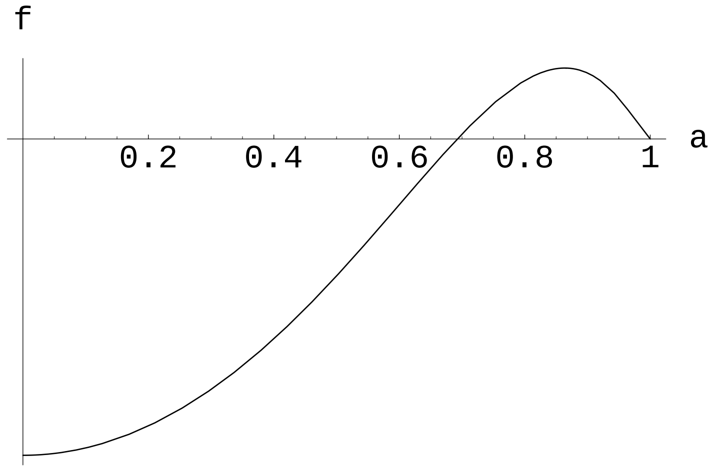

Figure 2: $f \equiv S^{\text{con.}}_{ren.} L^2/(T_0 X_2)$ as a function of a. One can see that for $a < a_c$, the renormalized action is negative; while for $a > a_c$, the renormalized action is positive.

For the connected membrane solution, the action is:

$$S^{\text{con.}}(a) = 2T_2 R^3 T_0 X_2 \int_0^{y_m} \frac{1}{y^3}\sqrt{\frac{y_m^6(1-\epsilon^6 y^6)}{y_m^6 - y^6}}dy. \tag{2.22}$$

As in [12, 35], We should subtract the action of two straight membranes stretched between the boundary ($y = 0$) and the horizon ($y = y_0 \equiv 1/\epsilon$).[6] These membranes extend in the y, t, x_2 directions and are just the disconnected membrane configuration. After the subtraction, the action is

$$\begin{aligned} S^{\text{con.}}_{ren.}(a) &= 2T_2 R^3 T_0 X_2 \left(\int_0^{y_m} \frac{1}{y^3}\sqrt{\frac{y_m^6(1-\epsilon^6 y^6)}{y_m^6 - y^6}}dy - \int_0^{y_0}\frac{dy}{y^3}\right) \\ &= \frac{4NT_0X_2}{\pi}\epsilon^2\left[\frac{1}{a^2}\int_0^1\left(\frac{1}{z^3}\sqrt{\frac{1-a^6z^6}{1-z^6}} - \frac{1}{z^3}\right)dz + \frac{1}{2}\left(1-\frac{1}{a^2}\right)\right]. \\ &= \frac{2NT_0X_2}{\pi}\epsilon^2\left(1 + \frac{\sqrt{\pi}\Gamma(-1/3)}{3a^2\Gamma(1/6)}\,{}_2F_1\left(-\frac{1}{2}, -\frac{1}{3}, \frac{1}{6}, a^2\right)\right) \end{aligned} \tag{2.23}$$

The dimensionless combination $S^{\text{con.}}_{ren.} L^2/(T_0 X_2)$ is the following function of a:

$$S^{\text{con.}}_{ren.} L^2/(T_0X_2) = \frac{8\pi N}{9}(LT_H)^2\left(1 + \frac{\sqrt{\pi}\Gamma(-1/3)}{3a^2\Gamma(1/6)}\,{}_2F_1\left(-\frac{1}{2}, -\frac{1}{3}, \frac{1}{6}, a^2\right)\right), \tag{2.24}$$

where $(LT_H)^2$ is given in eq. (2.21). This function is plotted in figure 2.

After eliminating a, we can obtain the functional relation between these two dimensionless combinations: $S^{\text{con.}}_{ren.} L^2/(T_0 X_2)$ and LT_H. This functional relation is plotted in figure 3.

[6] This action represents the masses of these two strings.

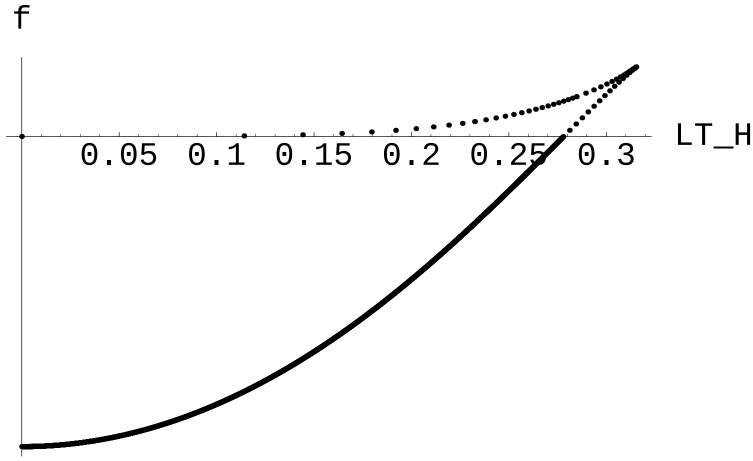

Figure 3: The functional relation between $f \equiv S^{\text{con.}}_{ren.}(a)L^2/(T_0X_2)$ and LT_H. The upper dashed curve is the one corresponding to the membrane configuration with the smaller a, i. e., a_2; The lower solid-dashed curve is the one corresponding to the membrane configuration with the smaller a, i. e., a_1. For $L < L_c$, the solid curve gives us VL^2/X_2 as a function of LT_H, where V is the interaction potential per length between these two strings, while for $L > L_c$, the interaction potential is zero. We note that this result is at the leading order of large N expansion.

As to the disconnected membrane configuration, this solution always exists for any L. Due to our substraction prescription, the renormalized action vanishes:

$$S^{\text{con.}}_{ren.}(a) = 0. \tag{2.25}$$

One can see from figure 2 that there is a value $a_c \approx 0.692$, such that $S^{\text{con.}}_{ren.}(a_c) = 0$. One can also see that

$$S^{\text{con.}}_{ren.}(a) \begin{cases} < 0, \text{ when } a < a_c; \\ > 0, \text{ when } a > a_c. \end{cases} \tag{2.26}$$

We can also find from figure 1 that for any $L < L_{\text{max.}}$, $a_2(L) > a_c$.

So for any given $L < L_{\text{max.}}$, among the renormalized action of three possible membrane configurations, $S^{\text{con.}}_{ren.}(a_1(L))$, $S^{\text{con.}}_{ren.}(a_2(L))$, and $S^{\text{dis.}}_{ren.}(L)$, the smallest one is[7]

$$\begin{cases} S^{\text{con.}}_{ren.}(a_1(L)), \text{ when } a \leq a_c; \\ S^{\text{dis.}}_{ren.}(L) = 0, \text{ when } a \geq a_c. \end{cases} \tag{2.27}$$

While for any $L > L_{\text{max.}}$, the only possible membrane configuration is the disconnected one whose renormalized action vanishes.

So at the leading order of large N expansion, the interaction potential per length between two infinite strings is:

$$\frac{V}{X_2} = \begin{cases} S^{\text{con.}}_{ren.}(a_1(L))/(X_2T_0), & \text{for } L \leq L_c; \\ 0 & \text{for } L \geq L_c. \end{cases} \tag{2.28}$$

[7]Figure 3 tells us that for any $L < L_{\max}$, we always have $S^{\text{con.}}_{ren.}(a_1(L)) < S^{\text{con.}}_{ren.}(a_2(L))$.

Here L_c ($\approx 0.278/T_H$) is the value of L such that $a_1(L_c) = a_c$. One can see from figure 1 and figure 2 that $L_c < L_{\max.}$. For $L < L_c$, the functional relation between the dimensionless combination VL^2/X_2 and LT_H is given by the solid curve in figure 3. Physically, we can take L_c as the screening length. When $L \le L_c$, the two strings can interact with each other. And when $L \ge L_c$, the two strings are screened by the thermal fluctuation and de-associate.

Now we further study the potential in the case of $LT_H \ll 1$. In this case, we have $a \ll 1$. Under this condition, we can expand eq. (2.21) in powers of a,

$$LT_H = \frac{3c}{\pi} a \left(1 - \frac{3}{14} a^6 - \frac{75}{728} a^{12} + \cdots\right), \tag{2.29}$$

where

$$c \equiv \frac{\sqrt{\pi}\Gamma(\frac{2}{3})}{\Gamma(\frac{1}{6})}. \tag{2.30}$$

Then we can get

$$a = t(1 + \frac{3}{14} t^6 + \frac{309}{728} t^{12} + \cdots), \tag{2.31}$$

where $t \equiv \pi LT_H/(3c)$.

From this result and the expansion of eq. (2.24) in powers of a, we get

$$\frac{V}{X_2} = \frac{N}{L^2}\left(-\frac{8c^3}{\pi} + \frac{8\pi}{9}(LT_H)^2 - \frac{32\pi^5}{5103c^3}(LT_H)^6 + \cdots\right). \tag{2.32}$$

Take $T_H \to 0$ in above equation, we arrive at the zero temperature result in [17]:

$$V/X_2 = -\frac{8\sqrt{\pi}\Gamma(\frac{2}{3})^3 N}{\Gamma(\frac{1}{6})^3 L^2}, \tag{2.33}$$

which is always lower than the finite temperature results.

In summary, the asymptotic behavior of our results at finite temperature is: when $L \ll 1/T_H$, V/X_2 goes like $1/L^2$ similar to what happens at zero temperature; while when $L \gg 1/T_H$, the potential is zero since the interaction is screened by the finite temperature effects.

From the above discussion, one can also see that VL^2/X_2 depends on T_H only through the combination $T_H L$, this is due to the underlying conformal symmetry although this symmetry is broken at finite temperature.

3. M5-brane description

In this section, we turn to study the M5-brane description of the straight Wilson-Polyakov surface operator. We expect that this description should be a better one when the Wilson-Polyakov surface is in higher dimensional representations, like what happens in the zero-temperature case [26].

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Let us first give a brief review of the M5-brane covariant equations of motion in an eleven-dimensional curved spacetime [42]. We are only interested in the bosonic components of the equations, which include the scalar equation and the tensor equation. The scalar equation takes the form

$$G^{mn}\nabla_m \mathcal{E}_n^{\underline{c}} = \frac{Q}{\sqrt{-g}}\epsilon^{m_1\cdots m_6}\left(\frac{1}{6!}H^{\underline{a}}{}_{m_1\cdots m_6} + \frac{1}{(3!)^2}H^{\underline{a}}{}_{m_1 m_2 m_3}H_{m_4 m_5 m_6}\right)P_{\underline{a}}{}^{\underline{c}} \tag{3.1}$$

and the tensor equation is of the form

$$G^{mn}\nabla_m H_{npq} = Q^{-1}(4Y - 2(mY + Ym) + mYm)_{pq}. \tag{3.2}$$

The various quantities in the above equations of motion are introduced as follows. There exist a self-dual 3-form field strength h_{mnp} on the M5-brane worldvolume, from which, one can define

$$k_m{}^n = h_{mpq}h^{npq}, \tag{3.3}$$

$$Q = 1 - \frac{2}{3}\mathrm{Tr}k^2, \tag{3.4}$$

$$m_p{}^q = \delta_p{}^q - 2k_p{}^q, \tag{3.5}$$

$$H_{mnp} = 4Q^{-1}(1+2k)_m^{\ q}h_{qnp} \tag{3.6}$$

Note that h_{mnp} is self-dual with respect to worldvolume metric but not H_{mnp}, which instead satisfies a nonlinearly self-dual condition and also the Bianchi identity

$$dH_3 = -\underline{H}_4 \tag{3.7}$$

where $\underline{H}_4$ is the pull-back of the target space 4-form flux. The induced metric is simply

$$g_{mn} = \mathcal{E}_m^{\underline{a}}\mathcal{E}_n^{\underline{b}}\eta_{\underline{ab}} \tag{3.8}$$

where

$$\mathcal{E}_m^{\underline{a}} = \partial_m z^{\underline{m}} E_{\underline{m}}^{\underline{a}}. \tag{3.9}$$

Here $z^{\underline{m}}$ is the target spacetime coordinate, which is a function of worldvolume coordinate ξ through embedding, and $E_{\underline{m}}^{\underline{a}}$ is the component of target space vielbein. However, it is not g_{mn} but instead another tensor

$$G^{mn} = \left(1 + \frac{2}{3}k^2\right)g^{mn} - 4k^{mn}, \tag{3.10}$$

which appear in (3.1). And the covariant derivative in (3.1) means

$$\nabla_m \mathcal{E}_n^{\underline{c}} = \partial_m \mathcal{E}_n^{\underline{c}} - \Gamma^p_{mn}\mathcal{E}_p^{\underline{c}} + \mathcal{E}_m^{\underline{a}}\mathcal{E}_n^{\underline{b}}\omega_{\underline{ab}}^{\underline{c}} \tag{3.11}$$

where Γ^p_{mn} is the Christoffel symbol with respect to the induced worldvolume metric and $\omega_{\underline{ab}}^{\underline{c}}$ is the spin connection of the background spacetime. Also one has

$$P_{\underline{a}}{}^{\underline{c}} = \delta_{\underline{a}}^{\underline{c}} - \mathcal{E}_{\underline{a}}^m \mathcal{E}_m{}^{\underline{c}}. \tag{3.12}$$

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Moreover, there is a 4-form field strength $H_{\underline{a}_1\cdots\underline{a}_4}$ and its Hodge dual 7-form field strength $H_{\underline{a}_1\cdots\underline{a}_7}$:

$$\begin{aligned} H_4 &= dC_3 \\ H_7 &= dC_6 + \frac{1}{2}C_3 \wedge H_4 \end{aligned} \tag{3.13}$$

The frame indices on H_4 and H_7 in the scalar and the tensor equations have been converted to worldvolume indices with factors of $\mathcal{E}_m^{\underline{c}}$. From them, we can define

$$Y_{mn} = [4 \star \underline{H} - 2(m \star \underline{H} + \star\underline{H}m) + m \star \underline{H}m]_{mn}, \tag{3.14}$$

where

$$\star\underline{H}^{mn} = \frac{1}{4!\sqrt{-g}}\epsilon^{mnpqrs}\underline{H}_{pqrs}. \tag{3.15}$$

These two quantities appear in the tensor equation of motion.

These equations of motion can be obtained from the non-chiral action [43, 44] or the PST (Pasti-Sorokin-Tonin) action [28, 29]. In the non-chiral action, a nonlinear self-dual condition for H_3 should be put by hand instead of coming from the variation of the action. This is similar to what happens in the case of ten-dimensional type IIB supergravity where the self-dual condition for 5-form field strength is put by hand. In the PST action, an auxiliary field is introduced to deal with the self-duality of H_3. We postpone a brief introduction of the PST action to the subsection 3.3, since only there this action is needed.

3.1 M5-brane configuration in $Sch.\text{-}AdS_7$

First we consider the M5-brane solution which is completely embedded in the $Sch.\text{-}AdS_7$ part of the background metric. In this case, we expect that due to the membrane interaction in the presence of background 4-form flux, the membrane will polarize to a M5-brane by blowing up an S^3 in the transverse direction. This is really the case for the Wilson surface operators discussed in [26]. Now we choose the coordinates of $Sch.\text{-}AdS_7$ such that the metric takes the following form:

$$ds^2 = \frac{R^2}{y^2}\Big(- f dt^2 + \frac{dy^2}{f} + dx^2 + dr^2 + r^2 d\Omega_3^2\Big). \tag{3.16}$$

In the case of the straight Wilson-Polyakov surface, let the worldvolume coordinates of M5-brane be $\xi_i, \quad i = 0, \cdots, 5$, and the embedding be

$$\xi_0 = t, \quad \xi_1 = x, \quad \xi_2 = y, \quad r = g(y), \tag{3.17}$$

$$\xi_3 = \alpha, \quad \xi_4 = \beta, \quad \xi_5 = \gamma, \tag{3.18}$$

where α, β, γ are the angular coordinates of S^3. This embedding is reasonable from the experience in the study of the Wilson surface operators. The induced metric is then

$$\begin{aligned} ds^2_{\text{ind}} &= \frac{R^2}{y^2}(-f d\xi_0^2 + d\xi_1^2 + (f^{-1} + g'^2)d\xi_2^2 + g^2 d\Omega_3^2) \\ &= \frac{R^2}{y^2}(-f dt^2 + dx^2 + (f^{-1} + g'^2)dr^2) + \frac{R^2 g^2}{y^2}(d\alpha^2 + \sin^2\alpha d\beta^2 + \sin^2\alpha\sin^2\beta d\gamma^2) \end{aligned} \tag{3.19}$$

where the prime denotes the derivative with respect to y. Without causing confusion, we simply let $t, x, y, \alpha, \beta, \gamma$ be the coordinates of the M5-brane worldvolume.

There is a self-dual 3-form field strength in the M5-brane worldvolume. Let us assume it to be

$$h_3 = \frac{a}{2}(1 + \star_{\text{ind}})\sqrt{\det G}d\alpha \wedge d\beta \wedge d\gamma \tag{3.20}$$

where a could be a function of y and $\det G$ is the determinant of the metric of S^3. In our case, we have

$$h_3 = \frac{a}{2}\left(\frac{R}{y}\right)^3 (g^3 \sin^2 \alpha \sin \beta d\alpha \wedge d\beta \wedge d\gamma + \sqrt{1 + fg'^2}dt \wedge dx \wedge dy). \tag{3.21}$$

Then we can calculate the relevant quantities k^{mn}, G^{mn} etc... It turns out that the physical 3-form field strength is

$$H_3 = 2a\left(\frac{R}{y}\right)^3 \left(\frac{\sqrt{1 + fg'^2}}{1 + a^2}dt \wedge dx \wedge dy + \frac{g^3}{1 - a^2}\sin^2 \alpha \sin \beta d\alpha \wedge d\beta \wedge d\gamma\right) \tag{3.22}$$

Since there is no pull-back of bulk 4-form field strength on the M5-brane worldvolume, we have $dH_3 = 0$, which gives the constraint

$$\frac{a}{1 - a^2}\frac{g^3}{y^3} = \text{constant} \tag{3.23}$$

The equation of motion on the tensor H_{npq}, in this case, is

$$G^{mn}\nabla_m H_{npq} = 0. \tag{3.24}$$

Here ∇_m is the covariant derivative with respect to the induced metric. We list the detailed Levi-Civita connection in appendix. It is somehow surprising that the tensor equation give the same constraint (3.23). It is remarkable that (3.23) is independent of the form of f.

For the scalar equation of motion, it is more involved. In our case, we have

$$\begin{aligned} &\mathcal{E}_t^{\underline{0}} = \frac{R}{y}\sqrt{f}, \quad \mathcal{E}_x^{\underline{1}} = \frac{R}{y}, \quad \mathcal{E}_y^{\underline{2}} = \frac{R}{y\sqrt{f}}, \quad \mathcal{E}_y^{\underline{3}} = \frac{R}{y}g', \\ &\mathcal{E}_\alpha^{\underline{4}} = \frac{Rg}{y}, \quad \mathcal{E}_\beta^{\underline{5}} = \frac{Rg\sin\alpha}{y}, \quad \mathcal{E}_\gamma^{\underline{6}} = \frac{Rg\sin\alpha\sin\beta}{y}, \end{aligned} \tag{3.25}$$

where we have set the veilbein of AdS_7 part of the target spacetime as

$$\begin{aligned} &\hat{\theta}^0 = \frac{R}{y}\sqrt{f}dt, \quad \hat{\theta}^1 = \frac{R}{y}dx, \quad \hat{\theta}^2 = \frac{R}{y\sqrt{f}}dy, \quad \hat{\theta}^3 = \frac{R}{y}dr, \\ &\hat{\theta}^4 = \frac{Rr}{y}d\alpha, \quad \hat{\theta}^5 = \frac{Rr\sin\alpha}{y}d\beta, \quad \hat{\theta}^6 = \frac{Rr\sin\alpha\sin\beta}{y}d\gamma. \end{aligned} \tag{3.26}$$

The corresponding spin connection could be found in appendix. The straightforward calculation shows that

$$G^{mn}\nabla_m \mathcal{E}_n^{\underline{c}} = 0, \qquad \text{except } \underline{c} = \underline{2} \text{ or } \underline{3}. \tag{3.27}$$

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The nontrivial components come from $\underline{c} = \underline{2}$ or $\underline{3}$. The right hand side of the scalar equation of motion consists of the matrix $P_{\underline{a}}^{\underline{c}} = \delta_{\underline{a}}^{\underline{c}} - \mathcal{E}_{\underline{a}}^{m}\mathcal{E}_{m}^{\ \underline{c}}$, which has nonvanishing components

$$P_{\underline{a}}^{\ \underline{c}} = \begin{pmatrix} \frac{fg'^2}{1+fg'^2} & -\frac{\sqrt{f}g'}{1+fg'^2} \\ -\frac{\sqrt{f}g'}{1+fg'^2} & \frac{1}{1+fg'^2} \end{pmatrix}. \tag{3.28}$$

where $\underline{a}, \underline{c}$ take values $\underline{2}, \underline{3}$.

For the background flux, we have a dual 7-form field strength in AdS_7 part,

$$H_{\underline{01}\cdots\underline{6}} = \frac{6}{R} \tag{3.29}$$

Note that our convention is a little different from the literature by a factor 2 since we have rescaled the radius of AdS_7. On the right hand side of the scalar equation, only 7-form field strength contributes since the M5-brane worldvolume is embedded simply into AdS_7 and there is no induced 4-form field strength on it.

It turns out that the nontrivial components $\underline{c} = \underline{2}$ and $\underline{3}$ of the scalar equation of motion give the same constraint:

$$\begin{aligned} \frac{6(1-a^4)}{\sqrt{1+fg'^2}} &= (1+a^2)^2 \left\{ \frac{3fg'}{1+fg'^2} - \frac{1}{(1+fg'^2)^2} \left(\frac{yf'g'}{2}(2+fg'^2) + fyg'' \right) \right\} \\ &\quad +3(1-a^2)^2 \frac{1}{1+fg'^2} \left(fg' + \frac{y}{g} \right). \end{aligned} \tag{3.30}$$

When one takes $f = 1$ and $g = \kappa^{-1} y$, the above equation is just the one for the Wilson surface operator in the symmetric representation, which was discussed in [26]. Generically even when one takes $f = 1$, the equation (3.30) is quite hard to solve analytically. When one consider the *Sch.*-AdS_7 with a nonconstant f, even the existence of the solution is not an easy problem. In [36], it was showed that there are no D3-brane solutions with finite total action dual to Wilson-Polyakov loops in four dimensional $\mathcal{N} = 4$ super Yang-Mills theory at finite temperature. In the case at hand, we can not directly use their argument since the total action of M5-brane is still not well-defined due to the subtlety of the boundary terms and the conformal anomalies [45]. Here we would like to just discuss the existence of the solution of the above differential equation. Let us impose the following initial condition:

$$g(0) = c_1, \qquad g'(0) = c_2. \tag{3.31}$$

For the case of $c_1 = 0$, we have mentioned that this initial value problem has a solution $g = \kappa^{-1} y$ when $\epsilon = 0$. This will guarantee that for small enough (positive) ϵ, the above initial value problem will have a solution in a finite interval $[0, y_0(\epsilon)]$. In another word, when the temperature is low enough, the M5-brane solution dual to Wilson surface in symmetric representation in zero-temperature theory will only be deformed, not be destroyed. However, for the case of $c_1 \neq 0$, we find that this initial value problem has no solutions.[8]

[8] We would like to thank Antonio Ambrosetti and Jiayu Li for discussions and helps on the study of this ordinary differential equation.

3.2 M5-brane configuration in $Sch.$-$AdS_7 \times S^4$

Now let us consider another possibility. We consider the M5-brane solution with topology $\Sigma_3 \times S^3$. Now Σ_3 will be in $Sch.$-AdS_7 and S^3 in S^4. Let the worldvolume coordinates of M5-branes be ξ_i, $i = 0, \cdots 5$ and the embedding be

$$\begin{aligned} &\xi_0 = t, \quad \xi_1 = x, \quad \xi_2 = y, \quad r = g(y) \\ &\xi_3 = \zeta_2, \quad \xi_4 = \zeta_3, \quad \xi_5 = \zeta_4, \quad \zeta_1 = \zeta^0 \end{aligned} \tag{3.32}$$

where ζ_i are the angular coordinates of S^4. Here we let ζ_1 be fixed at a constant ζ^0. The induced metric is

$$\begin{aligned} ds^2_{\text{ind}} = &\frac{R^2}{y^2}(-f dt^2 + dx^2 + (f^{-1} + g'^2)dy^2) \\ &+\frac{R^2 \sin^2 \zeta^0}{4}(d\zeta_2^2 + \sin^2 \zeta_2 d\zeta_3^2 + \sin^2 \zeta_2 \sin^2 \zeta_3 d\zeta_4^2). \end{aligned} \tag{3.33}$$

In this case, we take the self-dual 3-form field strength on the M5-brane worldvolume to be

$$h_3 = \frac{1}{2} a R^3 \left(\frac{\sqrt{1 + f g'^2}}{y^3} dt \wedge dx \wedge dy + \frac{\sin^3 \zeta^0}{8} \sin^2 \zeta_2 \sin \zeta_3 d\zeta_2 \wedge d\zeta_3 \wedge d\zeta_4 \right) \tag{3.34}$$

where a could be a function of y.

Similar to the above cases, we can get k^{mn}, $k^2 = \frac{3}{2}a^4$ and $Q = 1 - a^4$. And the physical 3-form is

$$H_3 = 2aR^3 \left(\frac{\sqrt{1 + f g'^2}}{(1 + a^2) y^3} dt \wedge dx \wedge dy + \frac{\sin^3 \zeta^0}{8(1 - a^2)} \sin^2 \zeta_2 \sin \zeta_3 d\zeta_2 \wedge d\zeta_3 \wedge d\zeta_4 \right). \tag{3.35}$$

The condition that $dH_3 = 0$ requires that a is a constant.

It is straightforward to check if it is possible and under what condition if possible that the above ansatz satisfy the equations of motion. Since a is a constant, the tensor equation is satisfied. And from the scalar equation, for the trivial embedding in AdS_7 $r =$ constant and the nontrivial embedding in S^4, the discussion is parallel to the one in [26], we get

$$a = \frac{\pm 1 + \sin \zeta^0}{\cos \zeta^0}. \tag{3.36}$$

As for the nontrivial embedding in $Sch.$-AdS^7 part, it is somehow interesting. Firstly note that the R.H.S of scalar equation is always vanishing in this case due to the pull-back of the 4-form or dual 7-form field strength is zero. At the end, we have the following equation:

$$\frac{3 f g'}{1 + f g'^2} - \frac{1}{(1 + f g'^2)^2} \left(\frac{y f' g'}{2} \left(2 + f g'^2 \right) + f y g'' \right) = 0, \tag{3.37}$$

which can be cast into the form

$$g'' + \frac{f' g'}{2f}(2 + f g'^2) - \frac{3}{y} g'(1 + f g'^2) = 0. \tag{3.38}$$

Obviously when g is a constant, which means that the embedding in AdS_7 is trivial, the above equation is satisfied, no matter what f is. This means that in this case we always have a M5-solution once (3.36) holds, just as we expected. We propose here that the solution with $g = 0$ should be dual to a straight Wilson-Polyakov surface operator in higher dimensional antisymmetric representation.

Certainly it would be interesting to solve eq. (3.38). It looks simpler than the one for the symmetric case, but still hard to solve. For example, let $f = 1$, which reduce to the background without the Schwarzschild blackhole. The equation is reduced to

$$g'' - \frac{3}{y}g'(1 + g'^2) = 0. \tag{3.39}$$

It could be solved exactly:

$$\begin{aligned} g = c_1 - \frac{1}{2}c_0^{-1/3}\Bigg((3^{-1/4} - 3^{1/4})F\Big(\beta, \frac{1+\sqrt{3}}{2\sqrt{2}}\Big) \\ + 2\sqrt[4]{3}E\Big(\beta, \frac{1+\sqrt{3}}{2\sqrt{2}}\Big) - \frac{2\sqrt{1 - c_0^2y^6}}{\sqrt{3} + 1 - c_0^{2/3}y^2}\Bigg), \end{aligned} \tag{3.40}$$

where c_0 and c_1 are two integral constants with c_0 being non-negative, β is defined as

$$\beta = \arccos \frac{\sqrt{3} - 1 + c_0^{2/3}y^2}{\sqrt{3} + 1 - c_0^{2/3}y^2}, \tag{3.41}$$

and F and E are elliptic integrals of the first and second kind, respectively. In this solution y can only take the value between 0 and $c_0^{-1/3}$. Obviously g being a constant is a trivial embedding. And the special one with $g = 0$ corresponds to the Wilson surface operator in anti-symmetric representation. However it is remarkable that for the pure $AdS_7 \times S^4$ case, there actually exist a two-parameter class of M5-brane configuration, characterized by the integral constant c_0, c_1. The one with g being constant is the one with half supersymmetries. However, with f not being a constant, the equation (3.38) is hard to solve.

The key point in the above discussion is that the embeddings in AdS_7 and S^4 are independent.

3.3 A universal result

As a generalization of the M5-brane solutions corresponding to Wilson(-Polyakov) surfaces in antisymmetric representation found in [26] and the previous subsection, we will prove a universal result on a class of M5-brane solutions in this subsection. We consider M-theory on $M_7 \times S^4$ with four form fluxes filling in S^4. We assume that this background is the solution of the eleven dimensional supegravity and a good background of M-theory. If M-theory on this background is dual to a field theory on the boundary of M_7, we expect this universal result is useful to study the Wilson(-Polyakov) surface operators in the field theory on the boundary. We need not to require that this background has any supersymmetries. AdS_7 and $Sch.$-AdS_7 are two special examples of M_7.

The background metric on $M_7 \times S^4$ is

$$ds^2_{M_7\times S^4} = ds^2_{M_7} + \frac{R^2}{4}\left(d\zeta_1^2 + \sin^2\zeta_1 d\zeta_2^2 + \sin^2\zeta_1\sin^2\zeta_2 dz_3^2 + \sin^2\zeta_1^2\sin^2\zeta_2^2\sin^2\zeta_3^2 d\zeta_4^2\right). \tag{3.42}$$

We assume that there is a membrane solution in this background and the worldvolume of this membrane, Σ_3, is completely embedded in M_7 part of the background geometry. Locally we can always choose the coordinates of the worldvolume such that the induced metric takes the following diagonal form:

$$ds^2_{\Sigma_3} = g_{\xi_0\xi_0}d\xi_0 d\xi_0 + g_{\xi_1\xi_1}d\xi_1 d\xi_1 + g_{\xi_2\xi_2}d\xi_2 d\xi_2. \tag{3.43}$$

This worldvolume is a three-dimensional submanifold of M_7 with minimal volume.

Now, we plan to show that from this membrane solution, we can obtained a M5-brane solution whose worldvolume has topology $\Sigma_3 \times \tilde{S}^3$ with the same Σ_3 in M_7 and $\tilde{S}^3$ in S^4.

Since here M_7 is quite generic, it is not easy to search for the M5-brane solution using the covariant M5-brane equations of motion. So in our discussions here we will use the PST (Pasti-Sorokin-Tonin) action [28, 29] of the M5-brane as in [27]. The bosonic part of the PST action is the following:

$$S_{\rm PST} = T_5\int d^6x\left(\sqrt{-\det(g_{mn}+i\tilde{H}_{mn})} - \frac{\sqrt{-g}}{4}\tilde{H}^{mn}H_{mn}\right) - T_5\int Z_6, \tag{3.44}$$

where

$$Z_6 = \underline{C}_6 - \frac{1}{2}\underline{C}_3\wedge H_3, \tag{3.45}$$

and

$$T_5 = \frac{1}{(2\pi)^5 l_p^6}, \tag{3.46}$$

is the tension of the M5-brane. In the above action,

$$\tilde{H}^{mn} = (*H)^{mnp}v_p, \tag{3.47}$$
$$H^{mn} = H^{mnp}v_p. \tag{3.48}$$

H_{mnp} is the 3-form field strength in the worldvolume of the M5-brane:

$$H_3 = dA_2 - \underline{C}_3, \tag{3.49}$$

and v_p is defined by introducing an auxiliary field b:

$$v_p = \frac{\partial_p b}{\sqrt{g^{mn}\partial_m b\partial_n b}}. \tag{3.50}$$

This auxiliary scalar field b can be an arbitrary scalar with nonzero gradient. We have made the choice that the gradient of b is spacelike. The equation of motion of the auxiliary field b is not independent. It can be obtained as a consequence of the equation of motion of the 2-form gauge potential, which takes the following form after appropriate gauge fixing:

$$H_{mn} = \mathcal{V}_{mn}, \tag{3.51}$$

where

$$\mathcal{V}_{mn} = -\frac{2}{\sqrt{-g}} \frac{\delta\sqrt{-\det(g_{mn} + i\tilde{H}_{mn})}}{\delta \tilde{H}^{mn}}. \tag{3.52}$$

The relation (3.51) can be understood as a generalized non-linear self-dual condition.

The ansatz of our M5-brane solution is the following: as mentioned before, we take the Σ_3 part of the worldvolume to be the same as the worldvolume of the above membrane solution. The coordinates of this part are still chosen to be ξ_0, ξ_1, ξ_2. As to the $\tilde{S}^3$ part, we choose the worldvolume coordinates to be

$$\xi_3 = \zeta_2, \quad \xi_4 = \zeta_3, \quad \xi_5 = \zeta_4, \tag{3.53}$$

and we let ζ_1 to be fixed at ζ^0. We also make the following ansatz for dA_2:

$$dA_2 = \frac{R^3}{8}\tilde{a}\sin^2\zeta_2 \sin\zeta_3 d\zeta_2 \wedge d\zeta_3 \wedge d\zeta_4, \tag{3.54}$$

here $\tilde{a}$ is a constant. We choose the background three form gauge potential to be

$$C_3 = \frac{R^3}{8}(3\cos\zeta_1 - \cos^3\zeta_1)\sin^2\zeta_2 \sin\zeta_3 d\zeta_2 \wedge d\zeta_3 \wedge d\zeta_4. \tag{3.55}$$

so

$$\underline{C_3} = \frac{R^3}{8}(3\cos\zeta^0 - \cos^3\zeta^0)\sin^2\zeta_2 \sin\zeta_3 d\zeta_2 \wedge d\zeta_3 \wedge d\zeta_4. \tag{3.56}$$

From now on, we will define $d(\zeta^0)$ as

$$d(\zeta^0) \equiv 3\cos\zeta^0 - \cos^3\zeta^0, \tag{3.57}$$

then

$$H_3 = \frac{R^3}{8}(\tilde{a} - d(\zeta^0))\sin^2\zeta_2 \sin\zeta_3 d\zeta_2 \wedge d\zeta_3 \wedge d\zeta_4. \tag{3.58}$$

The hodge dual of H_3 is

$$*H = \frac{\sqrt{-\det g_{\Sigma_3}}(\tilde{a} - d(\zeta^0))}{\sin^3\zeta^0} d\xi_0 \wedge d\xi_1 \wedge d\xi_2. \tag{3.59}$$

We choose the auxiliary scalar field b to be ξ_2, then the only nonzero component of v^p is $v^{\xi_2} = \sqrt{g^{\xi_2\xi_2}}$. So the only nonzero independent compoent of $\tilde{H}_{mn}$ is

$$\tilde{H}_{\xi_0\xi_1} = \sqrt{-\det g_{\Sigma_3} g^{\xi_2\xi_2}} \frac{\tilde{a} - d(\zeta^0)}{\sin^3\zeta^0}. \tag{3.60}$$

Then the first term of the PST action is:

$$\begin{aligned} T_5 \int d^6\xi \sqrt{-\det(g + i\tilde{H})} = \frac{T_5 R^3}{8} \int d^6\xi \sqrt{-\det g_{\Sigma_3}} \\ \times \sin^2\zeta_2 \sin\zeta_3 \sqrt{\sin^6\zeta_0 + (\tilde{a} - d(\zeta^0))^2}. \end{aligned} \tag{3.61}$$

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It is easy to see that the second and the third terms of the PST action vanish for our ansatz. So the PST action for our ansatz is:

$$S_{\rm PST} = \frac{T_5R^3}{8}\int d^6\xi\sqrt{-{\rm det}g_{\Sigma_3}}\sin^2\zeta_2\sin\zeta_3\sqrt{\sin^6\zeta_0+(a-d(\zeta^0))^2}. \tag{3.62}$$

We need to find the value of ζ^0 such that the action take the minimal value. Define

$$x\equiv\cos\zeta^0, \tag{3.63}$$

and

$$f=\sin^6\zeta^0+(a-d(\zeta^0))^2=(1-x^2)^3+(\tilde{a}-3x+x^3)^2. \tag{3.64}$$

From $\frac{df}{dx}=0$, we get $x=\tilde{a}/2$.[9] Then

$$H=-\frac{R^3}{8}\cos\zeta^0\sin^2\zeta^0\sin^2\zeta_2\sin\zeta_3 d\zeta_2\wedge d\zeta_3\wedge d\zeta_4. \tag{3.65}$$

Now the action of M5-brane equal to the volume of Σ_3 times a constant. Then Σ_3 should be a 3-dimensional submanifold with minimal volume. It is guaranteed by the fact that Σ_3 is the worldvolume of a M2-brane whose configuration is the solution of the membrane equations of motion. So our ansatz does satisfy the M5-brane equations of motion when $\tilde{a}$ and ζ^0 satisfy

$$\cos\zeta^0=\frac{\tilde{a}}{2}. \tag{3.66}$$

Using eqs. (3.35) and (3.36), one can find that the $\tilde{S}^3$ part of H_3 of the M5-brane solution in the previous subsection is the same as the obtained H_3 in this section. This show that that M5-brane solution is a special case of the universal result of this section.[10] Another special case was studied in [26].

As a nontrivial check of this universal result, we have studied the following ansatz for membrane in $Sch.$-AdS_7 space:

$$\xi_0=t,\quad \xi_1=x,\quad \xi_2=y,\quad r=g(y). \tag{3.67}$$

This ansatz is just the $Sch.$-AdS_7 part of the M5-brane ansatz eq. (3.32) in the previous section. The membrane equations of motion for this ansatz give the same constraint on g as the one obtained from the M5-brane equations, eq. (3.38).

Using this universal result, one can easily obtained the M5-brane configurations corresponding to two parallel straight Wilson-Polyakov surfaces in the same higher anti-symmetric representation from the M2-brane configurations discussed in subsection 2.2.

[9] The other two solutions of $df/dx=0$, $x=\pm1$, will give us the M5-brane solutions with shrinking $\tilde{S}^3$. We will not consider these solutions here.

[10] In the PST formalism, the self-dual condition is eq. (3.51) which is from the equations of motion. We need not to ask H_3 to be constructed from a self-dual 3-form h_3 on the worldvolume of the M5-brane as what we did in the previous subsections. This is the reason why the H_3 in this subsection does not have a part along the directions in M_7.

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4. Conclusion and discussions

In this paper, we investigated the thermodynamical behaviors of six-dimension $(2,0)$ field theory by studying the Wilson-Polyakov surface operators in this theory. We proposed that these operators should be described by M-theory branes. When these operators are in low dimensional representations, M2-brane configuration is a good description. While if these operators are in higher dimensional representation, we suggested that a better description should be in terms of M5-branes. We used our membrane description to study the interaction potential between two strings and found that when the distance between them is small, the potential's behaviors are asymptotically similar to zero-temperature results [17], while if the distance is large enough the interaction will be screened by the finite temperature effects. Qualitatively this result is similar to the potential between two quarks in the four dimensional SYM [35].

Although the M5-brane solution dual to straight Wilson-Polyakov surfaces in anti-symmetric representations are not very hard to find. Searching for the M5-brane solution dual to the Wilson-Polyakov surfaces in symmetric representation leads to a quite complicated differential equation. We discussed the existence of the solution and showed that when the temperature is small enough, the M5-brane solution should exist.

Inspired by our study of M5-branes dual to Wilson-(Polyakov) surfaces, we proved a universal result on M5-brane solution in a quite generic background $M_7 \times S^4$ with four-form flux. Given any membrane solution in this background with worldvolume Σ_3 completely embedded in M_7, we get an M5-brane solution with topology $\Sigma_3 \times \tilde{S}^3$ with $\tilde{S}^3$ being in S^4. We hope that this universal result is useful to study the dynamics of M-theory branes in generic background noticing that supersymmetries play no roles at all here. We hope that this results will also be useful in probing some other six-dimensional theory which has a gravity dual.

Quite less is known about the six-dimensional superconformal field theory. This theory at finite temperature theory is even less studied. As mentioned in the introduction, by compacting on a two-torus, the six dimensional theory will reduced to four dimensional $\mathcal{N} = 4$ super Yang-Mills theory at low energy. If we wrapping the Wilson-Polyakov surface on a suitable circle of this two-torus, we expect to get the Wilson-Polyakov loop. Hope that this relation will tell us more about the thermodynamics of this six-dimensional theory in the future.

It would be interesting to study more thoroughly the properties of the six-dimensional superconformal field theory at the finite temperature. The theory could be in a phase of perfect fluid, just like the quark-gluon plasma phase of $\mathcal{N} = 4$ super-Yang-Mills theory at finite temperature. Then one can use the AdS gravity to study the physics in this phase [46].

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A. Various connections

In this appendix, we list various connections appeared in our calculation. For the induced metric (3.19), its Christoffel symbol has nonvanishing independent components:

$$\begin{aligned}
\Gamma^t_{yt} &= \frac{1}{2}\left(\frac{f'}{f}-\frac{2}{y}\right),\\
\Gamma^x_{yx} &= -\frac{1}{y},\\
\Gamma^y_{tt} &= \frac{f}{2(1+fg'^2)}\left(-\frac{2f}{y}+f'\right),\\
\Gamma^y_{xx} &= \frac{f}{(1+fg'^2)y},\\
\Gamma^y_{yy} &= -\frac{1}{y}+\frac{1}{1+fg'^2}\Big(-\frac{f'}{2f}+fg'g''\Big),\\
\Gamma^y_{\alpha\alpha} &= \frac{fg^2}{1+fg'^2}\left(\frac{1}{y}-\frac{g'}{g}\right),\\
\Gamma^y_{\beta\beta} &= \Gamma^y_{\alpha\alpha}\sin^2\alpha,\\
\Gamma^y_{\gamma\gamma} &= \Gamma^y_{\alpha\alpha}\sin^2\alpha\sin^2\beta,\\
\Gamma^\alpha_{y\alpha} &= \Gamma^\beta_{y\beta}=\Gamma^\gamma_{y\gamma}=\left(-\frac{1}{y}+\frac{g'}{g}\right),\\
\Gamma^\alpha_{\beta\beta} &= -\sin\alpha\cos\alpha,\\
\Gamma^\alpha_{\gamma\gamma} &= -\sin^2\beta\sin\alpha\cos\alpha,\\
\Gamma^\beta_{\alpha\beta} &= \Gamma^\gamma_{\alpha\gamma}=\frac{\cos\alpha}{\sin\alpha},\\
\Gamma^\beta_{\gamma\gamma} &= -\sin\beta\cos\beta,\\
\Gamma^\gamma_{\beta\gamma} &= \frac{\cos\beta}{\sin\beta}.
\end{aligned} \tag{A.1}$$

For the $Sch.\text{-}AdS_7$ spacetime, its nonvanishing independent components of spin connection are

$$\begin{aligned}
\omega^{\underline{2}}_{\underline{00}} &= \frac{y^2}{R}\partial_y\left(\frac{\sqrt{f}}{y}\right), \quad \omega^{\underline{2}}_{\underline{ii}}=\frac{\sqrt{f}}{R}, && \text{for } i\neq 0,2,\\
\omega^{\underline{3}}_{\underline{ii}} &= -\frac{y}{Rr}, && \text{for } i=4,5,6,\\
\omega^{\underline{4}}_{\underline{ii}} &= -\frac{y\cos\alpha}{Rr\sin\alpha}, && \text{for } i=5,6,\\
\omega^{\underline{5}}_{\underline{66}} &= -\frac{y\cos\beta}{Rr\sin\alpha\sin\beta}.
\end{aligned} \tag{A.2}$$

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And its nonvanishing independent components of Christoffel symbol are

$$
\begin{aligned}
\Gamma^{\underline{y}}_{\underline{yy}} &= -\frac{1}{y} - \frac{f'}{2f}, \\
\Gamma^{\underline{y}}_{\underline{tt}} &= \frac{1}{2} f f' - \frac{f^2}{y}, \\
\Gamma^{\underline{y}}_{\underline{x}_i \underline{x}_i} &= \frac{f}{y}, \qquad\qquad \text{for } i = 1, \cdots, 5, \\
\Gamma^{\underline{t}}_{\underline{yt}} &= -\frac{1}{y} + \frac{f'}{2f}, \\
\Gamma^{\underline{x}_i}_{\underline{y}\underline{x}_i} &= -\frac{1}{y}.
\end{aligned} \tag{A.3}
$$

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